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Games with Bilateral Contracts on Side Payments*

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Abstract

We analyze two-stage games where players may make binding offers of schemes for side payment acceptance (or rejection) as well as those for side payments before choosing actions. We find that any set of efficient actions is played on an equilibrium path of the two-stage game when such bilateral contracts on side payments are interdependent.

1 Introduction

Coase (1960) put forth an idea that if property rights are well-defined, and bargaining is costless, then rational agents playing a game with externalities should contract to come to an efficient point. Coase (1960) was not explicit about the type of agreements between agents that are necessary as a form of bargaining to reach efficiency, but the idea has been widely accepted by economists.[1]

Contrary to the widespread belief in the idea, Jackson and Wilkie (2000) pointed out that side contracting does not always lead to efficiency even when there are no transactions costs, complete information, and binding contracts. They studied games where agents may make binding offers of

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[1]Much of the recent contracting literature has focused on imperfections related to costs of contracting, asymmetric information, limited enforcement of contracts, and so forth. Anderlini and Felli (2001) provide a discussion on the relationship of that literature to failures of the Coase theorem.

strategy-contingent side payments before choosing actions, and found that if there are only two agents, the agents are not always able to come to an agreement that supports an efficient strategy profile as an equilibrium point of the game. What kind of contracts will agents need to reach efficiency generally?

According to Jackson and Wilkie (2000), if there are three or more players, each efficient strategy profile is played on an equilibrium path in the game with side payments. However, Jackson and Wilkie (2000) only focused on voluntarily offered side payments and assumed that such side payments would always be accepted by transferees. This assumption might be thought of as arbitrary since voluntarily offered side payments could be invalidated by spontaneous rejection to receive them. Moreover, the results of Jackson and Wilkie (2000) depended upon another assumption as well that there is no budget constraint with players' transfer. Thus the question proposed above seems to remain unanswered at all. What kind of bilateral contracts would lead to efficiency even when agents face budget constraint with their transfer, no matter what number of players there are? This is the question we address in this paper.

We are to analyze two-stage games where players may make binding offers of schemes for side payment acceptance (or rejection) as well as those for side payments before choosing actions. A side payment from a player, say 1, to another, say 2, is implemented if and only if 1 offers the payment and 2 accepts it. If 2 rejects, then 1's offer is not in effect, and the payoff for the transfer remains with 1. We will see that every efficient strategy profile is played on an equilibrium path of the two-stage game, no matter what number of players there are, when the bilateral side contracts (transfer and receipt schemes) are somehow interdependent. Moreover, we will reach a similar result even when equilibrium contracts are required to meet agents' budget constraint with their transfer.

In what follows, we explain the timing of the two-stage game and present the model of the underlying game (the second-stage game) in Section 2. We present several models of bilateral side contracts and show the corresponding results in Sections 3-5. Some concluding remarks appear in Section 6.

2 The Underlying Game

We consider two-stage games played as follows.

Stage 1: Each player announces a transfer function profile (transfer scheme) and a receipt function profile (transfer acceptance/rejection scheme), each of which is assumed to be binding.
Stage 2: Each player chooses an action.

The players are given by a set $N = \{1, \ldots, n\}$. A player i's finite pure strategy space in the second stage game is denoted by X_i, with $X = \times_i X_i$. Let $\Delta(X_i)$ denote the set of mixed strategies for i, and let $\Delta = \times_i \Delta(X_i)$. We denote by x_i, x, μ_i, and μ generic elements of X_i, X, $\Delta(X_i)$, and Δ respectively. For simplicity, we sometimes use x_i and x to denote μ_i and μ respectively that place probability one on x_i and x. A player i's payoffs in the second stage game are given by a von Neumann-Morgenstern utility function $v_i : X \to \mathbb{R}$.

3 Contracts without Interdependence

3.1 Model

Let us consider the case when the contracts (the promises in the first stage) are not interdependent. In this case each agent's transfer scheme does not depend on any other's transfer nor receipt scheme and each agent's receipt scheme does not depend on any other's transfer nor receipt scheme.

A transfer function profile announced by player i in the first stage is denoted by $t_i = \left(t_{i1}, \ldots, t_{i(i-1)}, t_{i(i+1)}, \ldots, t_{in}\right)$, where $t_{ij} : X \to \mathbb{R}_+$ represents i's promises to j as a function of actions chosen in the second stage. Let T be the set of all possible t_{ij}. Let $t = (t_1, \ldots, t_n)$. A transfer function profile $t_i = \left(t_{i1}, \ldots, t_{i(i-1)}, t_{i(i+1)}, \ldots, t_{in}\right)$ announced by player i meets his budget constraint if $\sum_{j \neq i} t_{ij}(x) \leq \max\{0, v_i(x)\}$ for all x. A profile $t = (t_1, \ldots, t_n)$ of transfer function profiles is called feasible if every t_i meets i's budget constraint.

A receipt function profile announced by player i in the first stage is denoted by $r_i = \left(r_{i1}, \ldots, r_{i(i-1)}, r_{i(i+1)}, \ldots, r_{in}\right)$, where $r_{ij} : X \to \{0, 1\}$ represents i's acceptance (1) or rejection (0) of transfer from j as a function of actions chosen in the second stage. Let $r = (r_1, \ldots, r_n)$.

Given a profile t of transfer function profiles and a profile r of receipt function profiles in the first stage, and a play x in the second stage game, the payoff U_i to player i becomes

$$U_i(x,t,r) = v_i(x) + \sum_{j \neq i} \left(r_{ij}(x) t_{ji}(x) - r_{ji}(x) t_{ij}(x) \right).$$

Given a profile t of transfer function profiles and a profile r of receipt function profiles in the first stage, and a play μ in the second stage game, the expected payoff EU_i to player i becomes

$$EU_i(\mu,t,r) = \sum_x \times_k \mu_k(x_k) \left(v_i(x) + \sum_{j \neq i} \left(r_{ij}(x) t_{ji}(x) - r_{ji}(x) t_{ij}(x) \right) \right).$$

Let $EU_i(\mu) = \sum_x \times_k \mu_k(x_k) v_i(x)$.

Let $NE(t,r)$ denote the set of (mixed) Nash equilibria of the second stage game given (t,r) in the first stage. Let NE represent the set of (mixed) Nash equilibria of the underlying game (the second stage game without side contracts).

A strategy profile $\mu \in \Delta$ of the second stage game together with a vector $\overline{u} \in \mathbb{R}^n$ of payoffs such that $\sum_i \overline{u}_i = \sum_i EU_i(\mu)$ is *supportable* if there exists a subgame perfect equilibrium of the two stage game where some t and some r are announced in the first stage and μ is played in the second stage on the equilibrium path, and $EU_i(\mu,t,r) = \overline{u}_i$.

A strategy profile $\mu \in \Delta$ of the second stage game together with a vector $\overline{u} \in \mathbb{R}^n$ of payoffs such that $\sum_i \overline{u}_i = \sum_i EU_i(\mu)$ is *feasibly supportable* if there exists a subgame perfect equilibrium of the two stage game where some feasible t and some r are announced in the first stage and μ is played in the second stage on the equilibrium path, and $EU_i(\mu,t,r) = \overline{u}_i$.

3.2 Analysis

There exists a case when some set of efficient actions maximizing the total payoff is not supportable with any payoff distribution even if there exists a pure equilibrium of the underlying game.

Observation 1. *There is a case where some $(\overline{x},\overline{u})$ such that $\sum_i \overline{u}_i = \sum_i v_i(\overline{x})$ and $\sum_i v_i(\overline{x}) \geq \sum_i v_i(x)$ for all x is not supportable even if there exists ${}_ix$ for all i such that ${}_ix \in NE$ and $v_i({}_ix) \leq \overline{u}_i$.*

Proof of Observation 1. Consider a two-player game of prisoners' dilemma. The payoffs are represented as follows.

	C	N
C	2, 2	-1, 4
N	4, -1	0, 0

Let 1 and 2 denote the row and the column players respectively. Consider $(\overline{x}, \overline{u}) = ((C, C), (a, b))$ with $a + b = 4$. It is easy to note $\sum_i \overline{u}_i = \sum_i v_i(\overline{x})$ and $\sum_i v_i(\overline{x}) \geq \sum_i v_i(x)$ for all x. Moreover, $(N, N) \in NE$ and $v_i(N, N) \leq \overline{u}_i$ for each i if $a \geq 0$ and $b \geq 0$.

Suppose $(\overline{x}, \overline{u}) = ((C, C), (a, b))$ is supportable. Then, there exists a subgame perfect equilibrium of the two stage game where some t and some r are announced in the first stage and $\overline{x}$ is played in the second stage on the equilibrium path, and $EU_i(\overline{x}, t, r) = \overline{u}_i$. Suppose $a \leq b$ or $a \leq 2$. Since $\overline{x} \in NE(t, r)$, $r_{21}(N, C) = 1$. Therefore, if 1 announces $\widehat{t}_1$ and $\widehat{r}_1$ instead of t_1 and r_1 such that $\widehat{t}_{12}(x) = \begin{cases} 1.5 & \text{if } x = (N, C) \\ 0 & \text{otherwise} \end{cases}$ and $\widehat{r}_{12}(x) = 0$ for all x, then $NE\left(\left(\widehat{t}_1, t_2\right), \left(\widehat{r}_1, r_2\right)\right) = \{(N, C)\}$ and his payoff after transfer amounts to 2.5, which is greater than $\overline{u}_1$. This contradicts the assertion $(\overline{x}, \overline{u}) = ((C, C), (a, b))$ is supportable. Even for the case when $a \geq b$, another contradiction will be similarly reached. ■

4 Interdependent Contracts

4.1 Model

Next, let us consider the case when the contracts are interdependent. In this case each agent's transfer scheme (indirectly) depends on the others' receipt schemes and each agent's receipt scheme depends on the others' transfer schemes.

A transfer function profile announced by player i in the first stage is denoted by $t_i = \left(t_{i1}, \ldots, t_{i(i-1)}, t_{i(i+1)}, \ldots, t_{in}\right)$, where $t_{ij} : X \times Z \to \mathbb{R}_+$ with $Z = \{0, 1\}$ represents i's promises to j as a function of actions chosen in the second stage and indicators 0 and 1. Indicator 0 means that according to the transfer and receipt schemes announced in the first stage, a player rejects transfer from some other. Indicator 1 means that according to the transfer and receipt schemes announced in the first stage, every player accepts transfer from any other.

Note that if $t_{ij}(x,z) = z\tau_{ij}(x)$ for some $\tau_{ij} : X \to \mathbb{R}_+$, then the transfer scheme becomes degenerate, or $t_{ij}(x,z) = 0$ for all x, unless every player accepts transfer from the others. That is, when players are expected to promise acceptance to each other, such transfer function can be sensitive to a player's deviation on the receipt scheme.

Let T be the set of all possible t_{ij}. Let $t = (t_1, \ldots, t_n)$. A transfer function profile $t_i = (t_{i1}, \ldots, t_{i(i-1)}, t_{i(i+1)}, \ldots, t_{in})$ announced by player i meets his budget constraint if $\sum_{j \neq i} t_{ij}(x,z) \leq \max\{0, v_i(x)\}$ for all x and all z. A profile $t = (t_1, \ldots, t_n)$ of transfer function profiles is called feasible if every t_i meets i's budget constraint.

A receipt function profile announced by player i in the first stage is denoted by $r_i = (r_{i1}, \ldots, r_{i(i-1)}, r_{i(i+1)}, \ldots, r_{in})$, where $r_{ij} : (T^{n-1})^n \to \{0,1\}$ represents i's acceptance (1) or rejection (0) of transfer from j as a function of profiles of transfer function profiles announced in the first stage. Let $r = (r_1, \ldots, r_n)$.

Given a profile t of transfer function profiles and a profile r of receipt function profiles in the first stage, and a play x in the second stage game, the payoff U_i to player i becomes

$$U_i(x,t,r) = v_i(x) + \sum_{j \neq i} \left(r_{ij}(t)\, t_{ji}(x, a(t,r)) - r_{ji}(t)\, t_{ij}(x, a(t,r))\right)$$

where $a(t,r) = \times_{i,j,i \neq j} r_{ij}(t)$.

Given a profile t of transfer function profiles and a profile r of receipt function profiles in the first stage, and a play μ in the second stage game, the expected payoff EU_i to player i becomes

$$EU_i(\mu,t,r) =$$
$$\sum_x \times_k \mu_k(x_k) \left(v_i(x) + \sum_{j \neq i} \left(r_{ij}(t)\, t_{ji}(x, a(t,r)) - r_{ji}(t)\, t_{ij}(x, a(t,r))\right)\right)$$

where $a(t,r) = \times_{i,j,i \neq j} r_{ij}(t)$. Let $EU_i(\mu) = \sum_x \times_k \mu_k(x_k)\, v_i(x)$.

The definitions of $NE(t,r)$, NE, supportability, and feasible supportability are literally the same as those for the contracts without interdependence (Section 3).

4.2 Analysis

Any set of efficient actions maximizing the total payoff is supportable with some payoff distribution.

Proposition 1. *$(\overline{x},\overline{u})$ such that $\sum_i \overline{u}_i = \sum_i v_i(\overline{x})$ is supportable if there exists ${}_i\mu$ for all i such that ${}_i\mu \in NE$ and $EU_i({}_i\mu) \leq \overline{u}_i$.*

Proof of Proposition 1. Suppose for $(\overline{x},\overline{u})$ with $\sum_i \overline{u}_i = \sum_i v_i(\overline{x})$, there exists ${}_i\mu$ for all i such that ${}_i\mu \in NE$ and $EU_i({}_i\mu) \leq \overline{u}_i$.

Let $\tau_{ij} : X \to \mathbb{R}_+$ be such that $\tau_{ij}(x) = 0$ for all $x \neq \overline{x}$, and $\overline{u}_i = v_i(\overline{x}) + \sum\limits_{j\neq i} (\tau_{ji}(\overline{x}) - \tau_{ij}(\overline{x}))$ where $\tau_{ij}(\overline{x}) > 0$ for some j implies $\tau_{ji}(\overline{x}) = 0$ for all j. Let $\overline{t}$ and $\overline{r}$ be as follows.

$$\overline{t}_{ij}(x,z) = \begin{cases} z\left(\tau_{ij}(x) + \dfrac{\max\left\{0, v_i(x) + \sum\limits_{j\neq i}(\tau_{ji}(x) - \tau_{ij}(x)) - \overline{u}_i\right\}}{n-1}\right) & \text{if } x = (x_{-i}, \overline{x}_i) \\ 0 & \text{otherwise} \end{cases}$$

$$\overline{r}_{ij}(t) = \begin{cases} 1 & \text{if } t = \overline{t} \\ 0 & \text{otherwise} \end{cases}$$

Consider the following strategy profile (μ, t, r).
(1) $(t,r) = (\overline{t},\overline{r})$;
(2) if $(t,r) = (\overline{t}, (\overline{r}_{-i}, \widehat{r}_i))$ for some i, where $\widehat{r}_{ij}(\overline{t}) = 1$ for all $j \neq i$, then $\mu = \overline{x}$;
(2-1) if $(t,r) = ((\overline{t}_{-i}, \widehat{t}_i), (\overline{r}_{-i}, \widehat{r}_i))$ for some i, where $\widehat{t}_i \neq \overline{t}_i$ or $\widehat{r}_i \neq \overline{r}_i$ such that $\widehat{r}_{ij}(\overline{t}) = 0$ for some j, then $\mu = {}_i\mu$;
(2-2) otherwise $\mu \in NE(t,r)$.

Note first that for all i, $\overline{x} \in NE(\overline{t}, (\overline{r}_{-i}, \widehat{r}_i))$ and $U_i(\overline{x}, \overline{t}, (\overline{r}_{-i}, \widehat{r}_i)) = \overline{u}_i$ if $\widehat{r}_{ij}(\overline{t}) = 1$ for all $j \neq i$.

Suppose $(t,r) = ((\overline{t}_{-i}, \widehat{t}_i), (\overline{r}_{-i}, \widehat{r}_i))$ for some i, where $\widehat{t}_i \neq \overline{t}_i$. If $\mu = \widetilde{\mu} = ({}_i\mu_{-j}, \widetilde{\mu}_j)$ for some j, then when $j \neq i$

$$\begin{aligned} & EU_j(\mu,t,r) \\ =& \sum_x \times_k \widetilde{\mu}_k(x_k)\left(v_j(x) + \sum_{k\neq j}(r_{jk}(t)\,t_{kj}(x, a(t,r)) - r_{kj}(t)\,t_{jk}(x, a(t,r)))\right) \\ =& \sum_x \times_k \widetilde{\mu}_k(x_k)\left(\begin{array}{c} v_j(x) + \sum\limits_{k\neq j, k\neq i}(r_{jk}(t)\,t_{kj}(x, a(t,r)) - r_{kj}(t)\,t_{jk}(x, a(t,r))) \\ + (r_{ji}(t)\,t_{ij}(x, a(t,r)) - r_{ij}(t)\,t_{ji}(x, a(t,r))) \end{array}\right) \end{aligned}$$

$$=\sum_x \times_k \widetilde{\mu}_k(x_k)\left(v_j(x)+\sum_{k\neq j,k\neq i}(0\cdot 0-0\cdot 0)+(0\cdot t_{ij}(x,a(t,r))-r_{ij}(t)\cdot 0)\right)$$
$$=\sum_x \times_k \widetilde{\mu}_k(x_k)v_j(x)=EU_j(\widetilde{\mu})\leq EU_j({}_i\mu),$$

and when $j=i$

$$EU_i(\mu,t,r)$$
$$=\sum_x \times_k \widetilde{\mu}_k(x_k)\left(v_i(x)+\sum_{k\neq i}(r_{ik}(t)t_{ki}(x,a(t,r))-r_{ki}(t)t_{ik}(x,a(t,r)))\right)$$
$$=\sum_x \times_k \widetilde{\mu}_k(x_k)\left(v_i(x)+\sum_{k\neq i}(r_{ik}(t)\cdot 0-0\cdot t_{ik}(x,a(t,r)))\right)$$
$$=\sum_x \times_k \widetilde{\mu}_k(x_k)v_i(x)=EU_i(\widetilde{\mu})\leq EU_i({}_i\mu)\leq \overline{u}_i.$$

Suppose $(t,r)=\left(\overline{t},(\overline{r}_{-i},\widehat{r}_i)\right)$ for some i, where $\widehat{r}_i\neq\overline{r}_i$ such that $\widehat{r}_{ij}\left(\overline{t}\right)=0$ for some j. If $\mu=\widetilde{\mu}=\left({}_i\mu_{-j},\widehat{\mu}_j\right)$ for some j, then when $j\neq i$

$$EU_j(\mu,t,r)$$
$$=\sum_x \times_k \widetilde{\mu}_k(x_k)\left(v_j(x)+\sum_{k\neq j}(r_{jk}(t)t_{kj}(x,a(t,r))-r_{kj}(t)t_{jk}(x,a(t,r)))\right)$$
$$=\sum_x \times_k \widetilde{\mu}_k(x_k)\left(\begin{array}{c} v_j(x)+\sum_{k\neq j,k\neq i}(r_{jk}(t)t_{kj}(x,a(t,r))-r_{kj}(t)t_{jk}(x,a(t,r)))\\ +(r_{ji}(t)t_{ij}(x,a(t,r))-r_{ij}(t)t_{ji}(x,a(t,r)))\end{array}\right)$$
$$=\sum_x \times_k \widetilde{\mu}_k(x_k)\left(v_j(x)+\sum_{k\neq j,k\neq i}(1\cdot 0-1\cdot 0)+(1\cdot 0-r_{ij}(t)\cdot 0)\right)$$
$$=\sum_x \times_k \widetilde{\mu}_k(x_k)v_j(x)=EU_j(\widetilde{\mu})\leq EU_j({}_i\mu),$$

and when $j=i$

$$EU_i(\mu,t,r)$$
$$=\sum_x \times_k \widetilde{\mu}_k(x_k)\left(v_i(x)+\sum_{k\neq i}(r_{ik}(t)t_{ki}(x,a(t,r))-r_{ki}(t)t_{ik}(x,a(t,r)))\right)$$
$$=\sum_x \times_k \widetilde{\mu}_k(x_k)\left(v_i(x)+\sum_{k\neq i}(r_{ik}(t)\cdot 0-1\cdot 0)\right)$$
$$=\sum_x \times_k \widetilde{\mu}_k(x_k)v_i(x)=EU_i(\widetilde{\mu})\leq EU_i({}_i\mu)\leq \overline{u}_i.$$

Thus, (1)-(2-2) constitutes a subgame perfect equilibrium where $\left(\overline{t},\overline{r}\right)$ is announced in the first stage and $\overline{x}$ is played in the second on the equilibrium path, and $U_i\left(\overline{x},\overline{t},\overline{r}\right)=\overline{u}_i$. ∎

Note that $\bar{t}$ in the proof of Proposition 1 is sure to be feasible when $\overline{u}_i \geq 0$ for all i. That is, any set of efficient actions maximizing the total payoff is feasibly supportable with some payoff distribution if there exists an equilibrium of the underlying game in which each player enjoys nonnegative payoff without side payments.

Proposition 2. *$(\overline{x}, \overline{u})$ such that $\sum_i \overline{u}_i = \sum_i v_i(\overline{x})$ and $\overline{u}_i \geq 0$ is feasibly supportable if there exists ${}_i\mu$ for all i such that ${}_i\mu \in NE$ and $EU_i({}_i\mu) \leq \overline{u}_i$.*

5 Weakly Interdependent Contracts

5.1 Model

Finally, let us consider the case when the contracts are weakly interdependent. In this case each agent's transfer scheme does not depend on any other's transfer nor receipt scheme while each agent's receipt scheme depends on the others' transfer schemes.

The definitions of transfer function profiles and their feasibility are the same as those for the contracts without interdependence (Section 3). The definitions of receipt function profiles are the same as those for the interdependent contracts (Section 4).

Given a profile t of transfer function profiles and a profile r of receipt function profiles in the first stage, and a play x in the second stage game, the payoff U_i to player i becomes

$$U_i(x, t, r) = v_i(x) + \sum_{j \neq i} (r_{ij}(t)\, t_{ji}(x) - r_{ji}(t)\, t_{ij}(x)).$$

Given a profile t of transfer function profiles and a profile r of receipt function profiles in the first stage, and a play μ in the second stage game, the expected payoff EU_i to player i becomes

$$EU_i(\mu, t, r) = \sum_x \times_k \mu_k(x_k) \left(v_i(x) + \sum_{j \neq i} (r_{ij}(t)\, t_{ji}(x) - r_{ji}(t)\, t_{ij}(x)) \right)$$

Let $EU_i(\mu) = \sum_x \times_k \mu_k(x_k)\, v_i(x)$.

The definitions of $NE(t, r)$, NE, supportability, and feasible supportability are literally the same as those for the contracts without interdependence (Section 3).

5.2 Analysis

When players are two, any set of efficient actions maximizing the total payoff is supportable with some payoff distribution if there exists a pure equilibrium of the underlying game.

Proposition 3. *Let $n = 2$. Then $(\overline{x}, \overline{u})$ such that $\sum_i \overline{u}_i = \sum_i v_i(\overline{x})$ and $\sum_i v_i(\overline{x}) \geq \sum_i v_i(x)$ for all x is supportable if there exists ${}_ix$ for all i such that ${}_ix \in NE$ and $v_i({}_ix) \leq \overline{u}_i$.*

Proof of Proposition 3. Suppose for $(\overline{x}, \overline{u})$ with $\sum_i \overline{u}_i = \sum_i v_i(\overline{x})$ and $\sum_i v_i(\overline{x}) \geq \sum_i v_i(x)$ for all x, there exists ${}_ix$ for all i such that ${}_ix \in NE$ and $v_i({}_ix) \leq \overline{u}_i$. Let $\overline{t}$ and $\overline{r}$ be as follows.

$$\overline{t}_i(x) = \begin{cases} \max\{0, v_i(x) - \overline{u}_i\} & \text{if } x_j = \overline{x}_j \\ 0 & \text{otherwise.} \end{cases}$$

$$\overline{r}_i(t) = \begin{cases} 1 & \text{if } t_j = \overline{t}_j \\ 0 & \text{otherwise} \end{cases}$$

Note $\overline{x} \in NE(\overline{t}, \overline{r})$ and $U_i(\overline{x}, \overline{t}, \overline{r}) = \overline{u}_i$.

Consider the following strategy profile (μ, t, r).

(1) $(t, r) = (\overline{t}, \overline{r})$;
(2) if $(t, r) = (\overline{t}, (\overline{r}_j, \widehat{r}_i))$ and $\widehat{r}_i(\overline{t}) = 1$, then $\mu = \overline{x}$;
(2-1) if $(t, r) = (\overline{t}, (\overline{r}_j, \widehat{r}_i))$ and $\widehat{r}_i(\overline{t}) = 0$, then $\mu \in \{{}_ix, ({}_ix_i, \overline{x}_j)\} \cap NE(t, r)$;
(2-2) if $(t, r) = ((\overline{t}_j, \widehat{t}_i), (\overline{r}_j, \widehat{r}_i))$, $\widehat{t}_i \neq \overline{t}_i$, and $\widehat{r}_i(\overline{t}_j, \widehat{t}_i) = 0$, then $\mu = {}_ix$;
(2-3) if $(t, r) = ((\overline{t}_j, \widehat{t}_i), (\overline{r}_j, \widehat{r}_i))$, $\widehat{t}_i \neq \overline{t}_i$, and $\widehat{r}_i(\overline{t}_j, \widehat{t}_i) = 1$, then $\mu \in \{{}_ix, ({}_ix_j, \overline{x}_i)\} \cap NE(t, r)$;
(2-4) otherwise $\mu \in NE(t, r)$.

Suppose $(t, r) = (\overline{t}, (\overline{r}_j, \widehat{r}_i))$ and $\widehat{r}_i(\overline{t}) = 1$ for some i. Then, $NE(t, r) = NE(\overline{t}, \overline{r})$. Hence $\overline{x} \in NE(t, r)$, and $U_i(\overline{x}, t, r) = U_i(\overline{x}, \overline{t}, \overline{r}) = \overline{u}_i$ in the subgame (2).

Suppose $(t, r) = (\overline{t}, (\overline{r}_j, \widehat{r}_i))$ and $\widehat{r}_i(\overline{t}) = 0$ for some i. If $\mu = x = ({}_ix_i, \widehat{x}_j)$, then

$$\begin{aligned} U_j(x, t, r) &= v_j(x) + (\overline{r}_j(\overline{t})\,\overline{t}_i(x) - \widehat{r}_i(\overline{t})\,\overline{t}_j(x)) \\ &= v_j(x) + (\overline{t}_i(x) - 0) \\ &= \begin{cases} v_j(x) + \max\{0, v_i(x) - \overline{u}_i\} & \text{if } x_j = \overline{x}_j \\ v_j(x) & \text{otherwise} \end{cases} \end{aligned}$$

while if $\mu = x = ({}_ix_j, \widehat{x}_i)$, then

$$U_i(x, t, r) = v_i(x) + (\widehat{r}_i(\overline{t})\,\overline{t}_j(x) - \overline{r}_j(\overline{t})\,\overline{t}_i(x))$$

$= v_i(x) + \left(0 - \bar{t}_i(x)\right)$
$= \begin{cases} v_i(x) - \max\{0, v_i(x) - \bar{u}_i\} & \text{if } x_j = \bar{x}_j \\ v_i(x) & \text{otherwise} \end{cases}$
$\le v_i(x) \le v_i({}_ix)$.
Hence, if ${}_ix \notin NE(t,r)$, then $U_j(({}_ix_i, \bar{x}_j), t, r) \ge U_j(({}_ix_i, x_j), t, r)$ for all x_j, and $\bar{t}_i({}_ix_i, \bar{x}_j) > 0$, which implies $U_i(({}_ix_i, \bar{x}_j), t, r) = \bar{u}_i \ge U_i((x_i, \bar{x}_j), t, r)$ for all x_i. That is, if ${}_ix \notin NE(t,r)$, then $({}_ix_i, \bar{x}_j) \in NE(t,r)$. Thus, $\{{}_ix, ({}_ix_i, \bar{x}_j)\} \cap NE(t,r) \ne \emptyset$, and for all x_j,
$U_i(({}_ix_i, x_j), t, r) \begin{cases} \le v_i({}_ix) \le \bar{u}_i & \text{if } \bar{t}_i({}_ix_i, x_j) = 0 \\ = \bar{u}_i & \text{if } \bar{t}_i({}_ix_i, x_j) > 0 \end{cases}$
in the subgame (2-1).

Suppose $(t,r) = \left(\left(\bar{t}_j, \hat{t}_i\right), (\bar{r}_j, \hat{r}_i)\right)$, $\hat{t}_i \ne \bar{t}_i$, and $\hat{r}_i\left(\bar{t}_j, \hat{t}_i\right) = 0$. Then, $NE(t,r) = NE$ since $\bar{r}_j\left(\bar{t}_j, \hat{t}_i\right) = 0$ as well. Hence ${}_ix \in NE(t,r)$, and $U_i({}_ix, t, r) = v_i({}_ix) \le \bar{u}_i$ in the subgame (2-2).

Suppose $(t,r) = \left(\left(\bar{t}_j, \hat{t}_i\right), (\bar{r}_j, \hat{r}_i)\right)$, $\hat{t}_i \ne \bar{t}_i$, and $\hat{r}_i\left(\bar{t}_j, \hat{t}_i\right) = 1$. If $\mu = x = ({}_ix_j, \hat{x}_i)$, then

$U_i(x,t,r) = v_i(x) + \left(\hat{r}_i\left(\bar{t}_j, \hat{t}_i\right)\bar{t}_j(x) - \bar{r}_j\left(\bar{t}_j, \hat{t}_i\right)\hat{t}_i(x)\right)$
$= v_i(x) + \left(\bar{t}_j(x) - 0\right)$
$= \begin{cases} v_i(x) + \max\{0, v_j(x) - \bar{u}_j\} & \text{if } \hat{x}_i = \bar{x}_i \\ v_i(x) & \text{otherwise} \end{cases}$
while if $\mu = x = ({}_ix_i, \hat{x}_j)$, then
$U_j(x,t,r) = v_j(x) + \left(\bar{r}_j\left(\bar{t}_j, \hat{t}_i\right)\hat{t}_i(x) - \hat{r}_i\left(\bar{t}_j, \hat{t}_i\right)\bar{t}_j(x)\right)$
$= v_j(x) + \left(0 - \bar{t}_j(x)\right)$
$\le v_j(x) \le v_j({}_ix)$.
Hence, if ${}_ix \notin NE(t,r)$, then $U_i(({}_ix_j, \bar{x}_i), t, r) \ge U_i(({}_ix_j, x_i), t, r)$ for all x_i, and $\bar{t}_j({}_ix_j, \bar{x}_i) > 0$, which implies $U_j(({}_ix_j, \bar{x}_i), t, r) = \bar{u}_j \ge U_j((x_j, \bar{x}_i), t, r)$ for all x_j. That is, if ${}_ix \notin NE(t,r)$, then $({}_ix_j, \bar{x}_i) \in NE(t,r)$. Thus, $\{{}_ix, ({}_ix_j, \bar{x}_i)\} \cap NE(t,r) \ne \emptyset$, and for all x_i,
$U_i(({}_ix_j, x_i), t, r) \begin{cases} \le v_i({}_ix) \le \bar{u}_i & \text{if } \bar{t}_j({}_ix_j, x_i) = 0 \\ \le \bar{u}_i & \text{if } \bar{t}_j({}_ix_j, x_i) > 0 \end{cases}$
in the subgame (2-3).

Thus, (1)-(2-4) constitutes a subgame perfect equilibrium where feasible $\bar{t}$ and $\bar{r}$ are announced in the first stage and $\bar{x}$ is played in the second stage on the equilibrium path, and $U_i\left(\bar{x}, \bar{t}, \bar{r}\right) = \bar{u}_i$. ∎

Note that $\bar{t}$ in the proof of Proposition 3 is sure to be feasible when $\overline{u}_i \geq 0$ for all i. That is, when players are two, any set of efficient actions maximizing the total payoff is feasibly supportable with some payoff distribution if there exists a pure equilibrium of the underlying game in which each player enjoys nonnegative payoff without side payments.

Proposition 4. *Let $n = 2$. Then $(\overline{x}, \overline{u})$ such that $\sum_i \overline{u}_i = \sum_i v_i(\overline{x})$, $\overline{u}_i \geq 0$, and $\sum_i v_i(\overline{x}) \geq \sum_i v_i(x)$ for all x is feasibly supportable if there exists ${}_ix$ for all i such that ${}_ix \in NE$ and $v_i({}_ix) \leq \overline{u}_i$.*

6 Concluding Remarks

We found that there is a class of (feasible) side contracts which may induce efficient equilibrium play in two-player games as well as in three-or-more-player games (Propositions 1 and 2). What to do next is to see whether the contracts proposed here are the simplest ones in the class. In fact there exist simpler (feasible) side contracts for two-player games (Propositions 3 and 4). We will find out whether three-or-more-player games also have such alternatives.

7 Reference

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